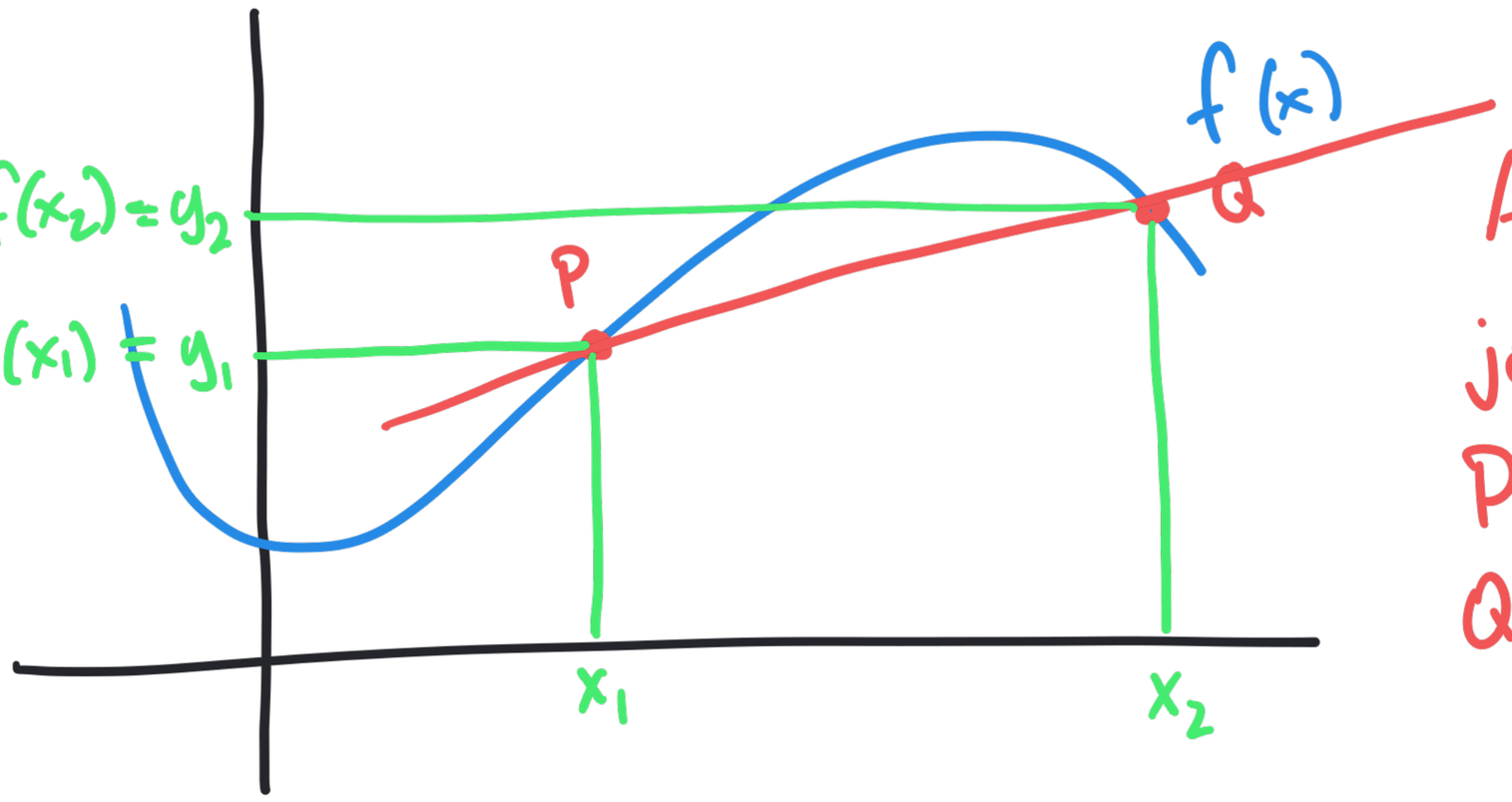
Intro Video: Section 2.1
secants, tangents, and velocity

Math F251X Calculus I

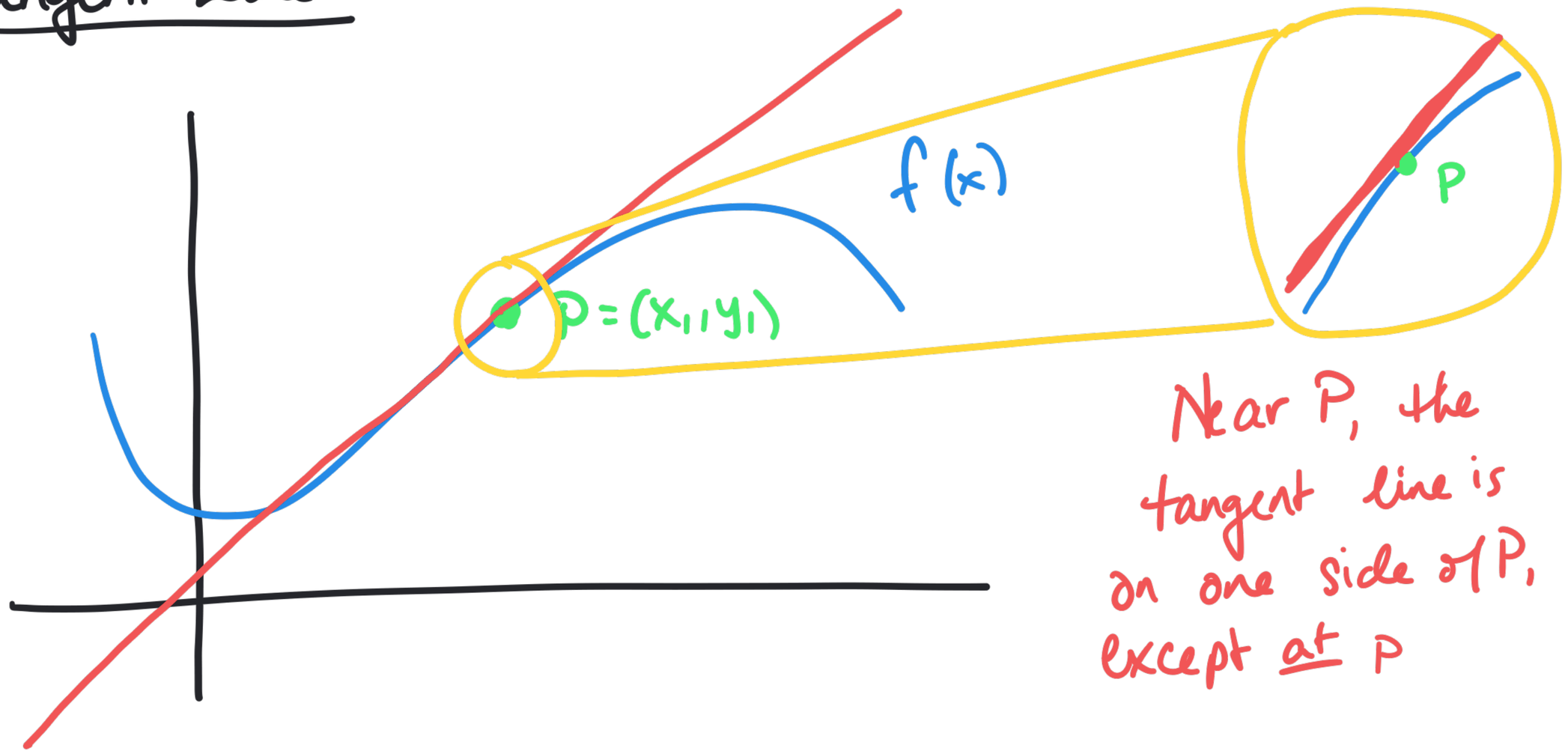
Secant Lines



A secant line joins two points $P = (x_1, y_1)$ and $Q = (x_2, y_2)$

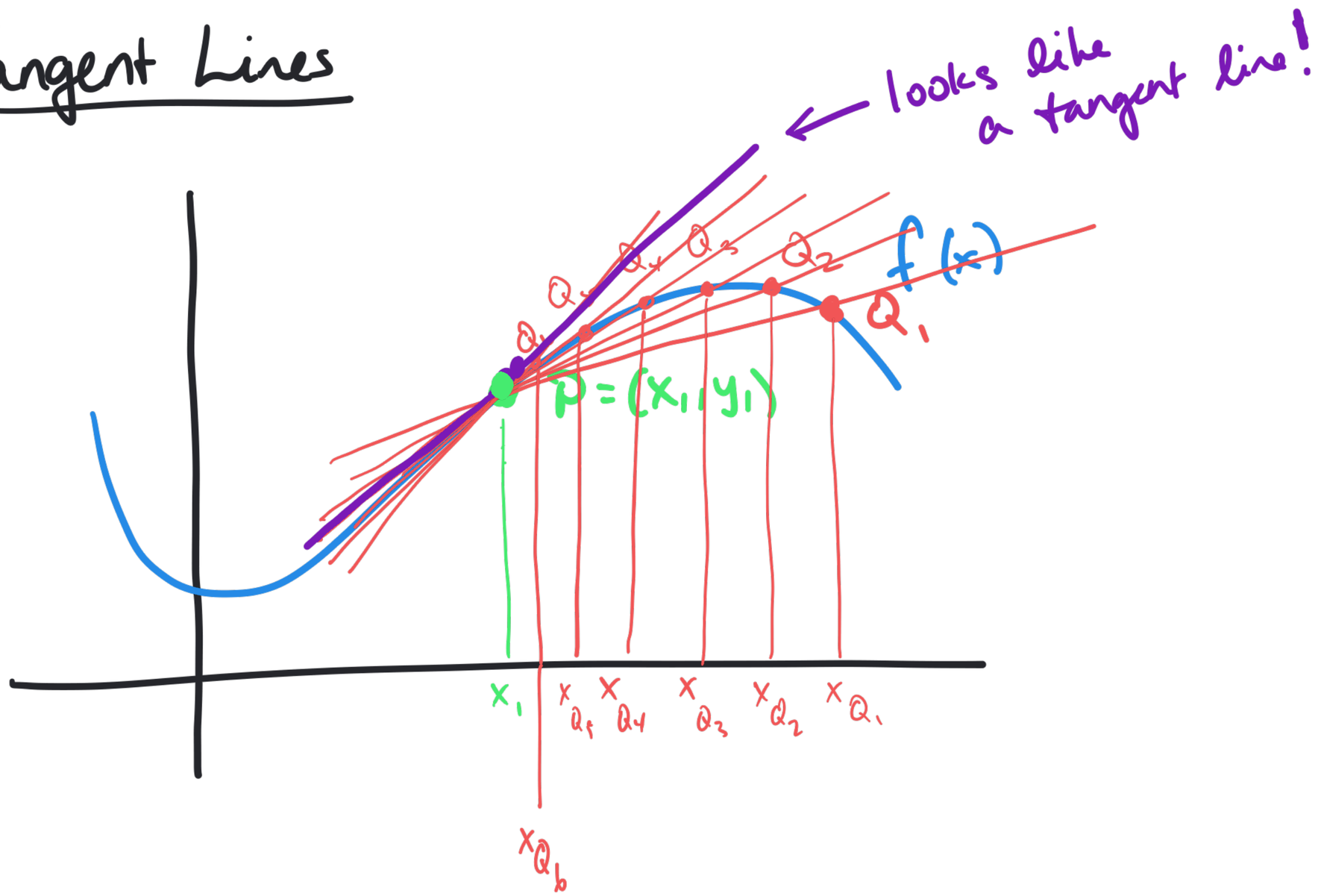
$$\text{Slope of secant line} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

Tangent Lines



Near P, the tangent line is on one side of P, except <u>at</u> P

Tangent Lines



Example : $f(x) = x^2$

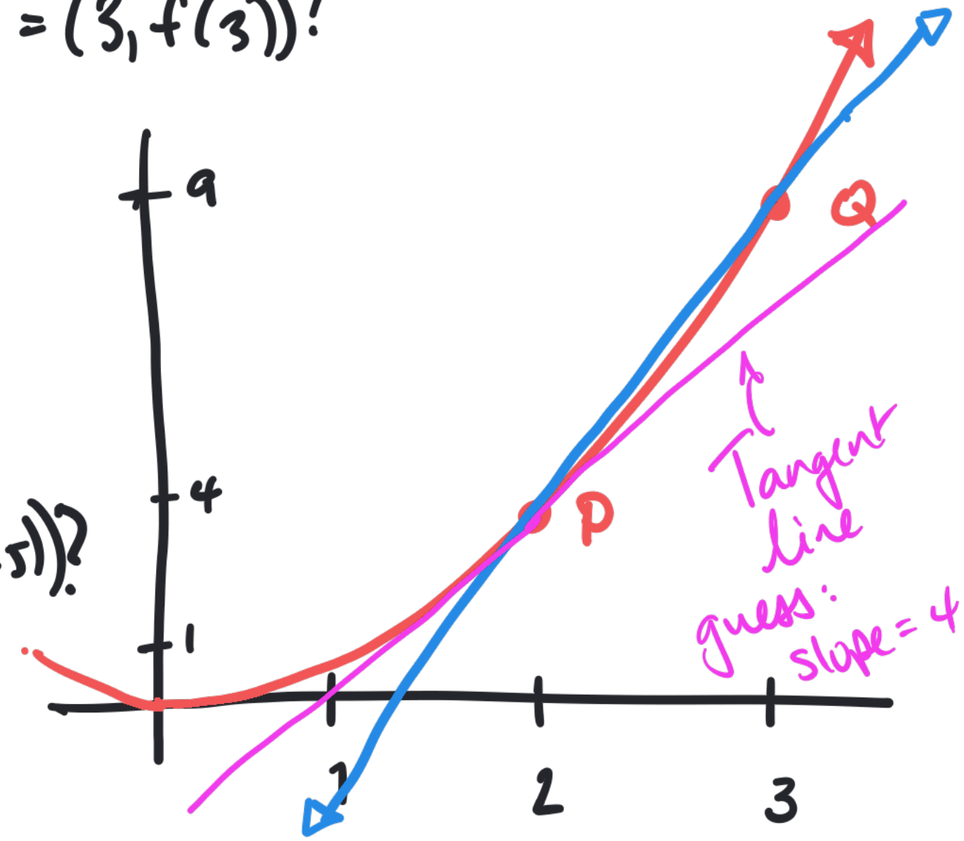
① What is the slope of the secant line between $P = (2, f(2))$ and $Q = (3, f(3))$?

$$\text{Slope} = \frac{f(3) - f(2)}{3 - 2} = \frac{9-4}{3-2} = \frac{5}{1} = 5.$$

② What is the slope of the secant line between $P = (2, f(2))$ and $(2.5, f(2.5))$?

$$\text{Slope} = \frac{f(2.5) - f(2)}{2.5 - 2} = \frac{6.25 - 4}{0.5} = \frac{2.25}{0.5}$$

$$= 4.5$$



Estimate Slope of Tangent Line at P: Let Q get close to P

x_Q	3	2.5	2.1	2.01	2.001	2.0001
y_Q	9	6.25	4.41	4.0401	4.004	4.0004
$\frac{\Delta y}{\Delta x} = \frac{y_Q - 4}{x_Q - 2}$	5	4.5	4.1	4.01	4.001	4.0001

So what?

- Suppose $f(x)$ measures the <u>distance</u> traveled in x seconds.

  Secant line measures $\frac{\text{Change in distance}}{\text{Change in time}}$

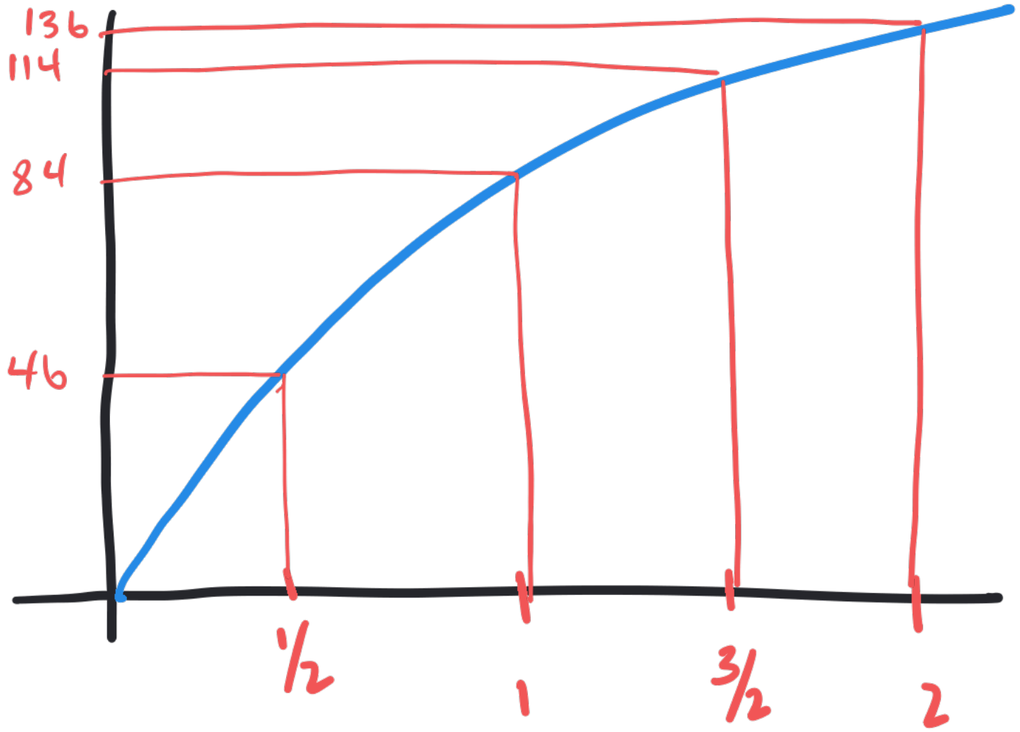
  Average velocity!

- Suppose $f(x)$ measures # of bacteria in a dish

  Secant line measures $\frac{\text{Change in \# of bacteria}}{\text{Change in time}}$

RATES OF CHANGE

Average velocity and instantaneous velocity



$s(t)$ ← position of an object, measured in centimeters, over a 2 minute interval

Average velocity over the interval $[\frac{1}{2}, 2]$?

$$\frac{\Delta \text{ distance}}{\Delta \text{ time}} = \frac{s(2) - s(\frac{1}{2})}{2 - \frac{1}{2}} = \frac{136 - 46}{2 - \frac{1}{2}} = \frac{90}{\frac{3}{2}} = \frac{180}{3} = 60$$

Average velocity over second minute? interval $[1, 2]$

$$\frac{\Delta \text{ distance}}{\Delta \text{ time}} = \frac{s(2) - s(1)}{2 - 1} = \frac{136 - 84}{2 - 1} = \frac{52}{1} = 52$$